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MASS-SPRING VS. FINITE ELEMENT MODELS OF ANISOTROPIC HEART VALVES: SPEED AND ACCURACY

Peter E. Hammer (1,2), Michael S. Sacks (3), Pedro J. del Nido (2), Robert D. Howe (4)

(1) Department of Biomedical Engineering
Tufts University
Medford, MA, USA

(2) Department of Cardiac Surgery
Children's Hospital
Boston, MA, USA

(3) Department of Bioengineering
University of Pittsburgh
Pittsburgh, PA, USA

(4) Harvard School of Engineering
and Applied Sciences
Cambridge, MA, USA

INTRODUCTION

Heat valve dysfunction can lead to heart failure and death, and surgery is the standard treatment. Valve repair surgery is performed under cardiopulmonary bypass making it difficult for the surgeon to know if a surgical modification will be effective when blood flow is restored. A surgical planning system has been proposed to improve surgical outcomes by allowing a surgeon to explore valve repair strategies on a computer model of a patient's valve (1). Many groups have published computational models of heart valves based on the finite element (FE) method, but they are prohibitively slow for simulating valve mechanics in an interactive setting. Mass-spring (MS) networks have been used as an alternative to FE methods for modeling deformable bodies, trading off accuracy for speed.

In this study, we assess the trade-off between speed and accuracy in an anisotropic MS model of aortic valve leaflets. We compare accuracy and computational cost of a MS model to a FE model of a membrane formulated for large deformations. We first compare stress-strain curves of simulated square patches of membrane under biaxial loading to stress-strain curves calculated directly from the constitutive law. Then, to assess accuracy in a way that is more relevant to heart valve loading, we simulate deformation of a semicircular membrane under typical pressure experienced by the aortic valve under peak load.

METHODS

A 15 mm square patch of aortic valve leaflet tissue with uniform thickness and density is meshed with unstructured triangles. Biomechanical behavior is approximated by a seven-parameter Fung exponential constitutive law which codifies the nonlinear, anisotropic behavior of aortic valve leaflet tissue. Parameters are determined by fit to biaxial test data using previously described methods (2). The mesh was modeled as a MS network by treating triangle edges as springs and distributing the mass of a triangle to its nodes (Fig. 1, inset). Spring constants are chosen based on mechanical properties using a method proposed by Van Gelder (3). Young's modulus in both principal directions was approximated using a bilinear function fit to the exponential constitutive law. Anisotropy is incorporated by computing Young's modulus for a given spring by weighting Young's modulus for the two principal directions by the orientation of the spring relative to the material fiber direction. During the simulation, the force in a given spring is computed based on its bilinear force-displacement relationship and applied to the two nodes bounding it. The net force on a node due to deformation is computed by summing the contributions of all springs sharing that node. A large-strain FE membrane model (4) is also used to compute internal forces on mesh nodes due to deformation and is used as the basis for comparison for speed and accuracy of the MS model.

Biaxial loading is simulated by applying in-plane tensile loads distributed along edges of the square patch and aligned perpendicular to the patch edges in the undeformed state. Five states of biaxial stress are simulated corresponding to peak biaxial Lagrangian stress ratios of 20:60, 30:60, 60:60, 60:30, and 60:20 kPa. Pressure loading is simulated by applying a constant pressure of 100 mmHg to a semicircular membrane with 20 mm diameter constrained along its semi-circumference; this approximates the size and shape of an aortic valve leaflet.

RESULTS

Plots of stress vs. strain for the MS and FE models are shown for simulated biaxial loading of a patch of tissue with fiber direction at 45^o to the patch edges (Fig 1). Average

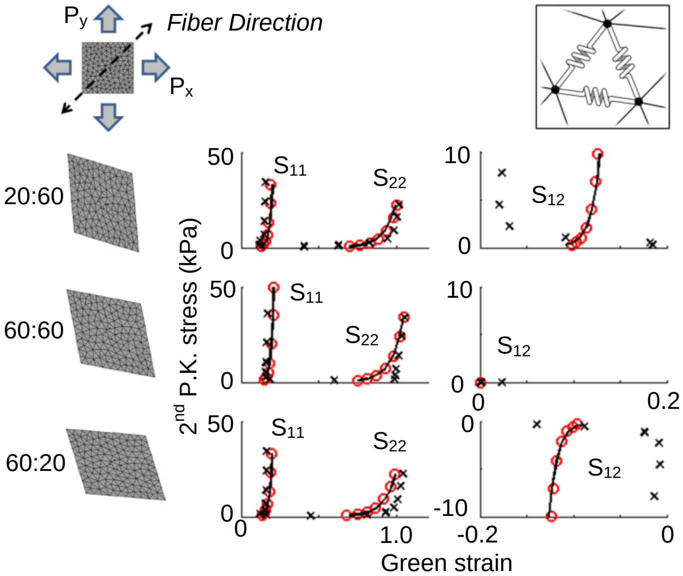

Fig. 1. For simulated biaxial tests, the square mesh in initial state and three loading states is shown on the left. On the right, 2nd Piola-Kirchhoff stress vs. Green strain is plotted for the same three loading states. Circles denote FE simulations, x's denote MS simulations, and solid curves are the constitutive equation.

magnitude of the error in strain for all points in all five loading curves was computed for the FE model and was 1.4% for the normal stress-strain curves and 2.2% for shear. For the MS model, the average magnitude of the error in strain was 9% for the normal stress-strain curves and 38% for shear.

The final deformed states of the semicircular patch simulating pressure loading are shown in Fig. 2. The images show the undeformed mesh, the final deformed state of the FE mesh, the final deformed state of the MS mesh, and a contour map of the distance between corresponding nodes on the deformed MS and FE meshes. Mean and maximum values for this distance are 0.30 and 0.98 mm, respectively. The length of the free edge of the mesh went from 20 mm in the undeformed state to 25.2 mm for the MS model and 26.4 mm for the FE model. Similarly, the midline of the mesh (in the cross-fiber direction) went from 10 mm in the undeformed state to 15.0 mm for the MS model and 14.4 mm for the FE model.

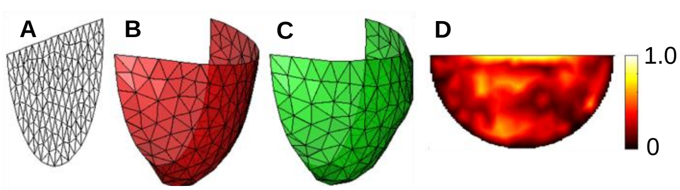

Fig. 2. Semicircular membrane loaded by 100 mmHg pressure. A) undeformed mesh, B) deformed FE model mesh, C) deformed MS model mesh, and D) contour plot showing distance in mm between final node positions predicted by FE and MS models.

The computational cost of the MS and FE methods was assessed both by counting the number of operations required to compute internal forces on the nodes of one triangle during one time step of the model and by measuring the time spent executing the portion of the program that performs these computations. Both measures of computational cost are summarized in Table 1.

Table 1. Speed and accuracy of MS and FE models.

	MS	FE
Computational cost		
Floating point operations	51	479
Time, re: FE (%)	10	(100)
Accuracy, biaxial loading		
Error, normal stress-strain (%)	9.0	1.4
Error, shear stress-strain (%)	38	2.2
Accuracy, pressure loading		
Free edge length (mm)	25.2	26.4
Centerline length (mm)	15.0	14.4
Mean surface error (mm)	0.30	-
Maximum surface error (mm)	0.98	-

DISCUSSION

The MS model examined here is approximately an order of magnitude faster than the FE model, a figure which agrees with published comparisons, but this comes at a cost of decreased accuracy, especially where shear behavior is concerned. This is not surprising because the MS model has no direct mechanism to control shear behavior. However, there is little shear loading on a pressurized aortic valve leaflet, so the MS model can approximate its deformation with small errors despite complex biomechanical properties of the leaflets. It is probably possible for the surgeon to incorporate errors of less than 1 mm into the factor of safety for a given surgical repair, thus the MS model is potentially valuable for realizing an interactive surgical planning system for heart valve repair.

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