

# The Sliding of Robot Fingers Under Combined Torsion and Shear Loading

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## Abstract

This paper is concerned with finding the magnitudes of applied moment and force which will cause a robot finger to slip on the surface of a grasped object. We first review friction and contact models used in previous grasp analyses, then describe an improved model which includes torsion-shear interaction. We present experimental measurements of the initiation of sliding as a function of loading. These measurements suggest that a simple linear function of torsion and shear magnitudes will adequately predict the onset of slip in many tasks. The use of this function is then illustrated in two measures of slip susceptibility for grasp planning.

## 1 Introduction

Sliding is an essential part of dextrous manipulation, and much current research on grasping is concerned with sliding. One direction for this research is the prediction of the maximum forces the finger-object contacts can sustain without slipping. Another direction is the use of controlled sliding of the fingers, such as humans often employ when manipulating objects [Mason 85, Peshkin 86, Fearing 86, Brock 87]. As part of our research on these issues, we are exploring the relationships between the factors that influence sliding. These factors include the kinematics and dynamics of each contact and the grasp as a whole, and material and object properties such as elasticity and surface texture. But in particular, an understanding of sliding requires realistic models of the frictional interaction between a finger and the gripped surface.

In this paper our goal is to find which combinations of forces and moments will cause a robot fingertip to slide on a grasped object. We begin with a review of models used in previous grasp analyses and note that they depend on many assumptions about the fingertip-object contact which are unrealistic. Kinematic models do not address dynamic aspects of contact behavior such as the interaction of torsion and shear. However, models which do analyze the physical interactions at the contact must make simplifying assumptions about material elasticity, linearity, isotropy, etc. We note the ubiquitous use of a speed-invariant Coulomb friction model and discuss the benefits that would accrue from the use of friction models more appropriate to elastomeric robot fingers. These benefits include the ability to model phenomena such as low-speed contact creep and stick-slip friction oscillations during sliding manipulation.

We then examine the interaction of torsion and shear in the commencement of sliding, and note that most previous grasping studies have failed to account for this coupling. We calculate the loadings at which the fingertip starts to slide for spherical "soft finger" and flat "very-soft finger" contact types.

Because these models of contacts are based on problematic assumptions, we have conducted experiments in which we have measured the onset of sliding for a range of applied torques and normal and tangential

forces. The data are in qualitative agreement with the improved model, but suggest a simpler stability criterion for future grasp analyses and planning. The use of this criterion is then applied to two measures of sliding for grasp stability prediction.

## 2 Models of Contact Behavior

In this section we review previous work on the behavior of robot finger contacts and discuss some of the common assumptions. The Coulomb friction model is compared to more accurate viscoelastic rubber friction models and ramifications of the improved model are described. The onset of sliding as a function of torsion and linear shear loading is examined, and calculations of the slipping limit for soft and very-soft contact types are presented.

### 2.1 Contact types and friction models

Several contact models are of practical interest in grasping and manipulation. In particular:

- **point** - exerts three force components and no moments; has negligible contact area, and therefore the Coulomb friction model is appropriate.
- **soft-finger** - exerts three force components and one moment about the local surface normal; has rounded (spherical) surface with finite contact area. Various friction models can be used. Additionally, the contact area varies with normal force.
- **very-soft-finger** - similar to the soft-finger contact, but planar with approximately uniform pressure distribution over a constant contact area. Various friction models can be used.

Most early work on grasping treated contacts from a purely kinematic viewpoint [e.g., Salisbury 85]. However, in order to determine a contact's resistance to torsion, the distribution of pressure across the contact must be known. Purely kinematic models say nothing about the distribution of pressure and therefore cannot deal with the relative magnitudes of linear and torsional friction.

Where the dynamics of the contact were considered, it was in terms of greatly simplified models. Earlier sliding analyses used the Coulomb friction model, in which friction is strictly proportional to normal force. This works well for contacts between materials like metals, but is not accurate for viscoelastic materials such as the rubbers with which most robot fingertips are covered. Cutkosky, Jourdain and Wright [1987] have described an improved friction model for elastomers in contact with smooth, dry surfaces which predicts that friction is proportional to the normal force to the  $2/3$ -power, and presented experimental confirmation of this model. This model shows that adhesion can greatly increase the friction limit at low normal loads.

Most workers also assume that the coefficient of friction is independent of sliding speed, although elastomeric friction is a strong function of speed [Moore 72]. If unmodeled, this can seriously impair the ability of a robot hand to perform controlled sliding operations. Furthermore, the viscoelastic nature of rubber friction implies the absence of true

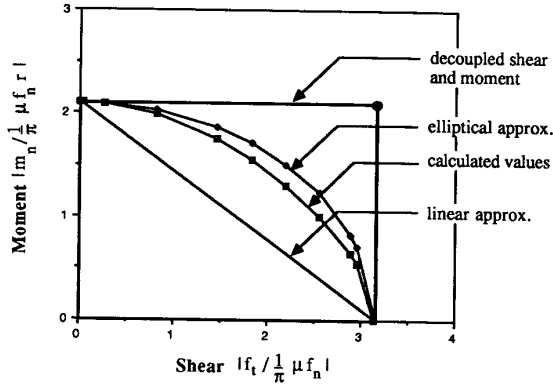


Figure 1: Various friction limits for a very-soft contact. Outer “box”: torsion and shear effects independent. Quasi-elliptical curve: calculated limits if torsion and shear effects interact. Ellipse and line: approximations to above; see Sect. 3.2.

static friction: most elastomers will “creep” when even a small tangential force or moment is applied. This can obviously lead to unexpected motion of the contact. Such non-Coulomb effects are important when selecting robot fingertip materials.

## 2.2 Torsion and linear shear limits

A common assumption in grasping analyses is that the friction limits due to torsion and shear are independent, so that the onset of slipping in rotation doesn’t depend on applied tangential load, and linear slipping doesn’t depend on applied moment (Figure 1). This approach continues to find widespread use despite its intuitive implausibility, because it results in constraint equations in which moment and linear force terms are uncoupled.

Jameson [1985] has developed an analysis which treats the combined effects of torsion and shear loading. He uses an indirect approach which assumes a linear elastic, spherical fingertip and uses Coulomb friction. The distribution of pressure across the contact is assumed to be Hertzian. The limits comprise a locus of points in the shear-moment plane, forming a quasi-elliptical curve.

We have performed a modified version of this calculation which includes the 2/3-power friction model for both soft and very-soft contacts. The results for the very-soft contact are shown in Figure 1; soft contact curves are similar. These limits are significantly lower than the limits for the independent torsion and shear assumption made in other grasping analyses.

The model used in these calculations is based on optimistic assumptions about the material (such as isotropy and linear elasticity), the pressure distributions (small strain assumption), and friction (Coulomb or 2/3-power friction model). Thus there is good reason to question the predictions of these analyses. We believe that the best way to answer these questions is to conduct experiments using realistic robot fingertip materials. In the next section we report on experiments in this direction.

## 3 Experimental Results

We have measured the onset of slipping as a function of applied moment and tangential force at several different normal forces. The results qualitatively confirm the expected relationships. However, significant differences between the theory and the data suggest a simpler, more conservative limit for grasp planning and execution.

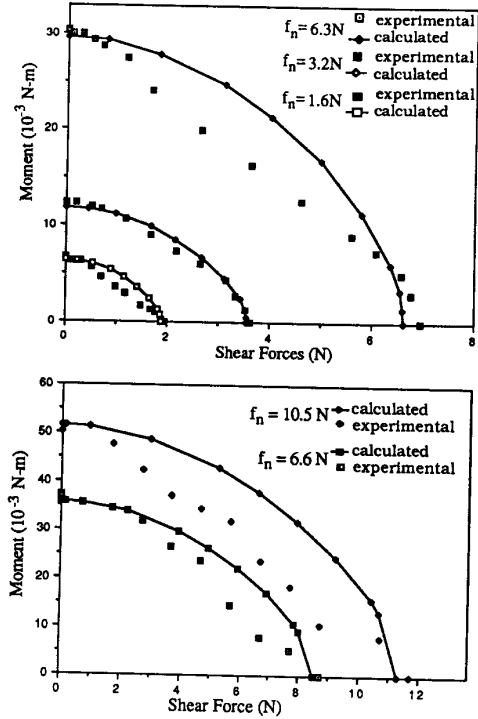


Figure 2: Measured contact friction limits. Unconnected points: experimental data; solid curves: calculated values. (a) Spherical contact (soft finger). (b) Flat contact (very-soft finger).

## 3.1 Apparatus and methods

The apparatus used for these measurements is the same one described by Cutkosky, Jourdain, and Wright [1987], modified to permit the application of a moment about the contact normal. An elastomeric fingertip is mounted at the center of a rotation stage that is in turn mounted on a lever. The lever presses the elastomer against a sliding platform with a polished glass surface, which permits visual observation of the contact behavior. The rotation stage and platform move on low-friction bearings, and the normal and tangential forces and the moment are controlled by adding and removing weights.

For each normal force, we used the following procedure. Starting with zero tangential force, the moment was increased by adding weights until slipping began; this value of the moment was recorded. Then the apparatus was reset, and the tangential force increased slightly. The moment was again increased from zero until the contact failed, and the loading conditions recorded. This sequence was repeated until slipping commenced due to the tangential force alone. Because of viscoelastic creep, the “start of slipping” was somewhat ambiguous. The criterion we used was the attainment of a (small) arbitrary sliding speed when the loads were applied at a fixed rate.

## 3.2 Results and discussion

Measurements were obtained at several normal forces for both a spherical section of an elastomer (a soft finger) and a flat, circular sample (a very-soft finger). The results are presented in Figure 2. The behavior of the soft and very-soft contacts is quite similar. At each normal force, the points fall somewhat below the near-elliptical form calculated in the preceding section, and far below the limit set by assuming that the effects of torsion and shear are independent. We also see that the points all fall above a line drawn between the maximum moment-zero shear point and the maximum shear-zero moment point (i.e., the points

where the data intercept the axes).

These results suggest two practical forms for the friction limits. If the prevention of slip is of paramount concern in a manipulation task, then a straight line drawn between the maximum moment and maximum shear is a simple, somewhat conservative estimate of the slipping limit. On the other hand, if controlled sliding is to play a role in the manipulation, an elliptical approximation would provide an improved estimate, especially near the axes where the slope of the limit curve deviates most from its average value.

Using the simpler of these forms, we propose a constraint equation for the initiation of sliding motion

$$f_t + A|m_n| \leq \mu|f_n| \quad (1)$$

where  $f_t = \sqrt{f_l^2 + f_m^2}$  is the tangential force on the contact,  $f_n$  is the normal force,  $m_n$  is the moment,  $\mu$  is the coefficient of friction, and  $A$  is a "proportionality constant" between the torsion and shear limits. In this linear approximation,  $A$  is equal to the ratio of the maximum moment to the maximum shear, and has units of inverse length. An elasticity analysis of the sort used by Jameson [1985] yields  $A \approx 2/r$ , where  $r$  is the contact radius. For the soft (spherical) contact which follows the Hertz pressure distribution,  $r \approx f_n^{1/3}$ , while the contact radius is by definition constant for the very-soft contact. The experimental values for  $A$  are within 50 percent of the theoretical values. For the soft finger, the variation of  $A$  with normal force follows the prediction of the elasticity analysis.

## 4 Measures of Slip for Grasp Analysis

As an example of combining torsion and shear criteria in predicting sliding behavior, we incorporate Equation (1) into two measures proposed by Kao and Cutkosky [1987]. The improved measures are substantially more accurate and suggest strategies for controlling the initiation of sliding.

### 4.1 Worst-case finger

We are interested in knowing when a finger will slip. Initially, suppose the grasping force at the contact is  $\mathbf{f}_p = [f_l \ f_m \ f_n \ m_n]^T$ , where  $l, m$  are coordinates in the plane tangent to the contact and  $n$  is normal to the contact.

When a new external force is applied to the object as part of the manipulation task or due to an unmodeled disturbance, the contact force changes and the new contact force becomes  $\mathbf{f}'_p = \mathbf{f}_p + \delta\mathbf{f}_p$ , where  $\delta\mathbf{f}_p = [\delta f_l \ \delta f_m \ \delta f_n \ \delta m_n]^T$ . Therefore we can rewrite Equation 1 as

$$\sqrt{(f_l + \delta f_l)^2 + (f_m + \delta f_m)^2} + A|m_n + \delta m_n| < \mu|f_n + \delta f_n| \quad (2)$$

For convenience, we can express this as a dimensionless ratio  $\alpha$  such that

$$\alpha_i = \frac{\sqrt{(f_l + \delta f_l)^2 + (f_m + \delta f_m)^2} + A|m_n + \delta m_n|}{\mu|f_n + \delta f_n|} \quad (3)$$

where the subscript  $i$  denotes the values for the  $i$ th finger. Note that  $\alpha \rightarrow 1$  as the finger starts to slide. For any given grasp, the finger closest to slipping (the "worst-case finger") is the finger for which  $\alpha_i$  is closest to unity.

Kao and Cutkosky [1987] show that the changes in contact force  $\delta\mathbf{f}_p$  can be calculated directly from grasp and finger stiffness matrices as a function of external forces. This permits us to infer several simple strategies for controlling the initiation of sliding. For example, if we decrease the stiffness of the worst-case finger then  $\alpha_i$  will decrease for that finger; this represents "relaxing" the finger. If we also increase the stiffness of the other fingers so that the overall grasp stiffness doesn't change too much, then we can usually stabilize the grasp.

## 4.2 Progression-toward-slipping

We can also look at the onset of slipping by constructing a potential function,  $V$ , such that

$$V = \sqrt{f_l^2 + f_m^2} + A|m_n| - \mu|f_n| \quad (4)$$

which indicates how far a finger is from its friction limits. But we would like to know not only how far a finger is from the friction limit, but also whether a given force applied to the grasped object will move it closer to or farther from the limit, and how fast. In other words, we would like to know the sensitivity of the finger to slipping when forces and moments are applied to the object.

Therefore, we define the change in the potential function, after an external force is applied, as

$$\delta V = (f'_l + A|m'_n| - \mu|f'_n|) - (f_l + A|m_n| - \mu|f_n|) \quad (5)$$

Expanding this and dropping small terms results in

$$\delta V = \frac{f_l \delta f_l + f_m \delta f_m}{\sqrt{f_l^2 + f_m^2}} + A(|m_n + \delta m_n| - |m_n|) - \mu \operatorname{sgn}(f_n) \delta f_n \quad (6)$$

where  $\operatorname{sgn}(f_n)$  is the sign of  $f_n$ .

$\delta V$  also suggests simple strategies for making a finger less likely to slip: (1) increase the normal force on the finger, and (2) ensure that  $\delta m_n$ ,  $\delta f_l$ , and  $\delta f_m$  are small or are such that they decrease the absolute values of  $m_n$ ,  $f_l$ , and  $f_m$ .

Kao and Cutkosky [1987] developed these measures for use with the three linear forces alone, but the moment has been easily incorporated into each of them. Comparison of the original measures with Equations (1) and (6) shows that only a simple moment term has been added in each case. Equation (1) could be used in many grasping analyses with little additional complexity and substantially increased accuracy.

## 5 Conclusions

Early work in grasping focused on the kinematics of finger-object contacts and adopted point-contact models with Coulomb friction. For soft fingertips, this leads to inaccurate results. Improved models account for finite contact areas and combined torsional and tangential loading, but these models also make unrealistic assumptions in modeling the fingertip material properties, friction, and pressure distributions. To test the validity of these models for practical manipulation tasks, we have experimentally measured the onset of slip for soft and very-soft fingertips made from elastomeric materials subjected to combined torsion and tangential shear loads. The results qualitatively confirm the predictions of the models, but also suggest a simpler, linear model. This model can be easily incorporated into many manipulation analyses while providing a significant increase in the accuracy of slip prediction.

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